Math 554 Review for Test 2.

I am leaving it up to you to know the basic definitions and the statements of the statements of the theorems we have covered to date. Here are some problems for practice. You should also go over our homework problems.

(1) If f is continuous on $(-\infty, \infty)$ and $f(2) = 5$ show that 2 has a neighborhood, N, so that $f(x) < 6$ for $x \in N$. *Hint:* Let $\varepsilon = 1$ in the definition of continuity.

(2) Show that the equation $2x + 3\cos(x^3) = 64$ has at least one solution.

(3) Show that $f(x) = x^2$ is not uniformly continuous on $[0, \infty)$. *Hint:* Let $\varepsilon = 1$. If $f(x)$ were uniformly continuous there would be a $\delta > 0$ so that $|f(x) - f(y)| < \varepsilon = 1$ whenever $|x - y| < \delta$. That is

$$|x^2 - y^2| = (x + y)|x - y| < \varepsilon = 1 \qquad \text{whenever} \qquad |x - y| < \delta.$$

What happens if x, y are very large? (Here the neaning of "very large" will depend on δ.)

(4) Problems 12, 13, and 15 on page 70 of the text.

(5) Give an example of a function where $|f|$ is continuous, but f is not continuous.

(6) (This on is a bit harder.) Let f be continuous on $[0, 1]$ and define

$$F(x) = \max f(t) : 0 \le t \le x.$$

(a) Explain why we can use "max" rather than "sup". *Hint:* Do we have any theorems that involve continuous functions on closed bounded intervals.

(b) Show that F is monotone increasing.

(c) Show that F is continuous.

(7) Various and sundry surprise mystery questions.