Mathematics 172 Test 3 Name: Key

Show your work to get credit.

1. (15 points) Two species of algae live in a aquarium. Let $x(t)$ be the population size, measured in grams of one of the species,after t days and $y(t)$ the population size of the other species measured in grams after t days. The competition between the two species is given modeled by the equations

$$\frac{dx}{dt} = .13x\left(\frac{50 - x - .8y}{50}\right)$$

$$\frac{dy}{dt} = .18y\left(\frac{75 - 2x - y}{75}\right)$$

(a) What are values of the following:

The carrying capacity of the x-species 50

The carrying capacity of the y-species 75

The intrinsic growth rate of the x species .13

The intrinsic growth rate of the y species .18

(b) If $x(3) = 45$ and $y(3) = 6$ compute the following

$x'(3) =$ .0234 $y'(3) =$ −.3024

$x'(3) = .13(45)(50-(45)-.8(6))/50$ | $y'(3) = .18(6)(75-2(45)-(6))/75$

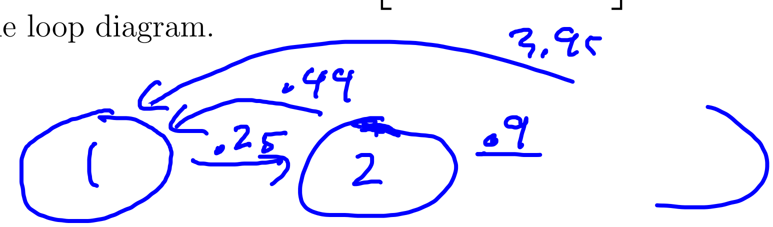
(c) Estimate the following

$x(3.5) \approx$ 45.0117 $y(3.5) \approx$ 5.8488

$x(3.5) \approx x(3) + x'(3)(.5) =$ $y'(3$ $3) + y'(3)(.5)$

2. (10 points) For the Leslie matrix $L = \begin{bmatrix} 0 & 0.44 & 3.95 \\ 0.25 & 0 & 0 \\ 0 & 0.9 & 0 \end{bmatrix}$

(a) Draw the loop diagram.

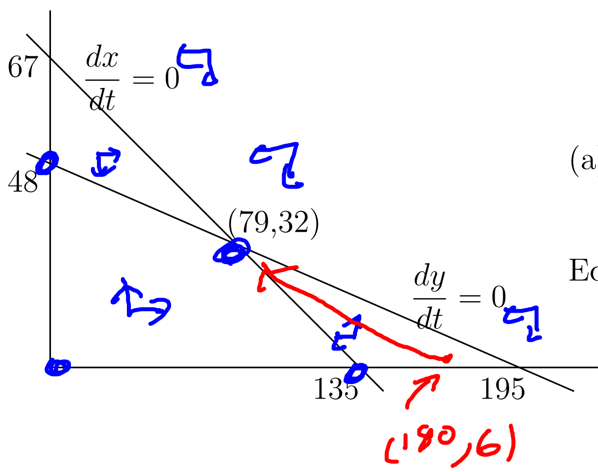


(b) If there are 100 in stage 2 this year, how many are there is stage 3 next year?

Number in stage 3 .9(100) = 90

3. (20 points) Consider populations of competing species modeled by the equations

$$\frac{dx}{dt} = r_1 x \left(\frac{K_1 - x - \alpha y}{K_1} \right)$$
$$\frac{dy}{dt} = r_2 y \left(\frac{K_2 - \beta x - y}{K_2} \right)$$



(a) If the phase space is as on the right:
(i) What are the equilibrium points?

Equilibrium points are (0,0), (135,0), (0,48), (79,32)

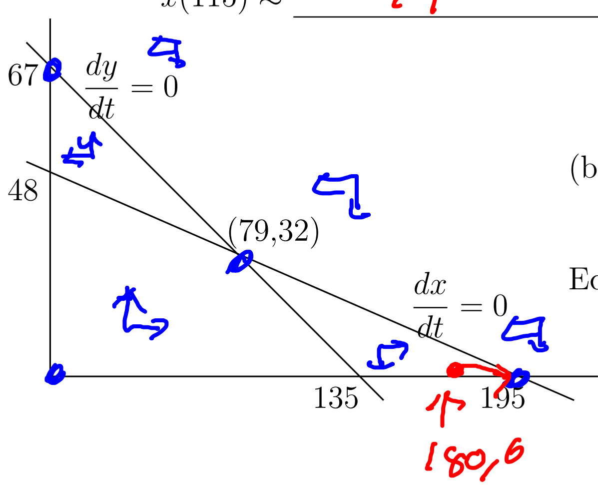
(ii) Draw in the arrows showing how points are moving.

(iii) What are the stable equilibrium points? Stable points are: (79, 32)

(iv) Circle one: ***Competitive coexistence*** ***Competitive exclusion***

(v) If $x(0) = 180$ and $y(0) = 6$ estimate the following

$x(115) \approx$ 79 $y(115) \approx$ 32



(b) If the phase space is as on the right:
(i) What are the equilibrium points?

Equilibrium points are (0,0), (195,0), (0,67), (79,32)

(ii) Draw in the arrows showing how points are moving.

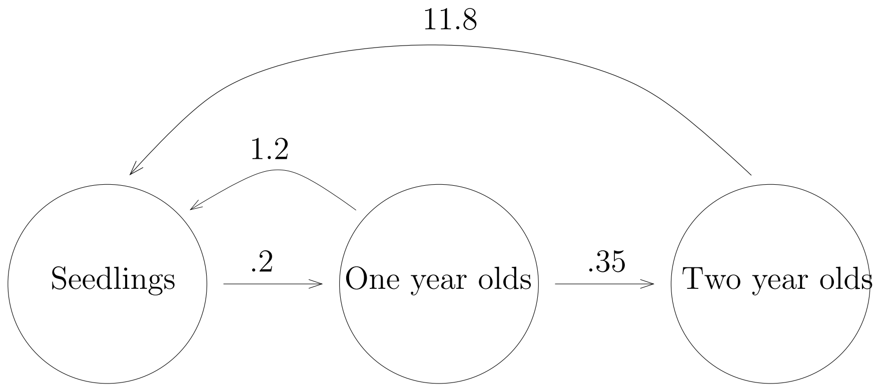
(iii) What are the stable equilibrium points? Stable points are: (195,0), (0,67)

(iv) Circle one: ***Competitive coexistence*** ***Competitive exclusion***

(v) If $x(0) = 180$ and $y(0) = 6$ estimate the following

$x(115) \approx$ 195 $y(115) \approx$ 0

4. (20 points) Parsley is a plant that lives for two years. We consider three stages. Stage 1: seedlings, Stage 2: one year olds, Stage 3: two year olds. The life history of a population of parsley in an abandoned garden is given by the following loop diagram.



(a) What is the Leslie matrix?

$$L = \begin{bmatrix} 0 & 1.2 & 11.8 \\ .2 & 0 & 0 \\ 0 & .35 & 0 \end{bmatrix}$$

(b) What does the number .2 mean?

This is the proportion of seedlings that survive to become one year olds

(c) What does the number 11.8 mean?

The average number of offspring to a two year old individual that lives to be a seedling

(d) What proportion of the seedlings live to be one year old?

The proportion is .2

(e) What proportion of the seedlings live to be two years old?

The proportion is $(.2)(.35) = .07$

(f) If this year there are 80 seedlings, 16 one year olds, and 6 two year olds, then after 20 years how many are in each stage and what proportion are in each stage?

Number in each stage:

Stage 1 131.66 Stage 2 25.868 Stage 3 8.876

Proportion in each stage:

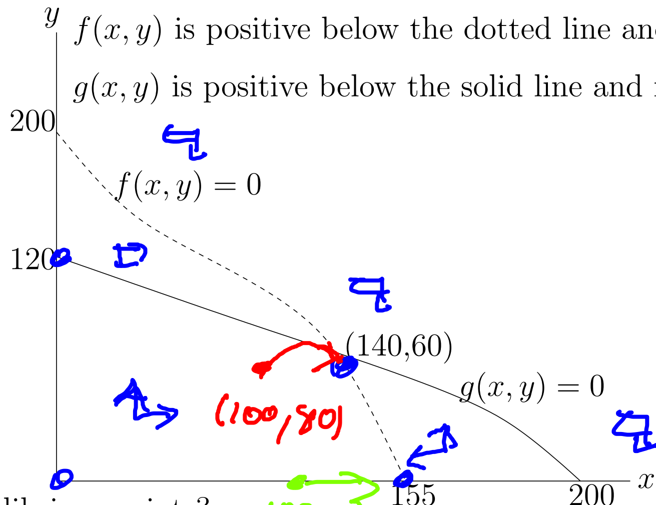
Stage 1 .791 Stage 2 .155 Stage 3 .053

5. (20 points) The x-species and y-species are competing for common resources. This competition is modeled by the following system of differential equations

$$\frac{dx}{dt} = xf(x,y)$$

$$\frac{dy}{dt} = yg(x,y)$$

The phase diagram of this system is given by



(a) What are the equilibrium points?

The equilibrium points are: (0,0), (155,0), (0,120), (145,60)

(b) Draw in the arrows showing which way points are moving in each of the regions of the phase diagram.

(c) What are the stable equilibrium points?

The stable equilibrium points are: (140, 60)

(d) If $x(0) = 100$ and $y(0) = 80$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 140 $y(100) \approx$ 60

(e) If $x(0) = 100$ and $y(0) = 0$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 155 $y(100) \approx$ 0

(f) If there are none of the y-species present, then what is the carrying capacity of the x-species?

x-carrying capacity is 155

(g) How would you describe this model? Circle one: ***completive coexistence, completive exclusion, x-species dominates, or y-species dominates.***

6. (15 points) A rain barrel has a population of yeast which is preyed on be a population of parameciums. Let $V(t)$ be the weight in grams of the total yeast population on day t and $P(t)$ the total weight of the parameciums in on day t. Model the growth of the two populations by the equations

$$\frac{dV}{dt} = 0.25V - 0.005VP$$
$$\frac{dP}{dt} = -.015P + 0.001VP$$

(a) What is the intrinsic growth rate of the yeast?

The growth rate is $.25$

(b) What is the death rate of the parameciums?

The death rate is $.015$

(c) Draw the phase space labeling the lines where $\frac{dV}{dt} = 0$ and $\frac{dP}{dt} = 0$ and put in arrows showing the directions that points are moving.

$$\frac{dV}{dt} = V(.25 - .005P). \qquad \hat{P} = \frac{.25}{.005} = 50$$

$$\frac{dP}{dt} = P(-.015 + .001V) \qquad \hat{V} = \frac{.015}{.001} = 15$$