# Mathematics 172

**Quiz #16** Name: Key

***You must show your work to get full credit.***

An South American rodent living on an island in the Amazon lives for three years and its life history is partly summarized by the Leslie matrix

$$A = \begin{bmatrix} 0.0 & 1.5 & 9.0 \\ 0.1 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 \end{bmatrix}$$

We wish to find the stable age distribution and the per capita growth rate. Assume that the stable age distribution is of the form

$$\mathbf{n} = \begin{bmatrix} 1 \\ v_2 \\ v_3 \end{bmatrix}.$$

Then if $\lambda = 1 + r$ we have that

$$A\mathbf{n} = \lambda\mathbf{n}.$$

where

$$\lambda\mathbf{n} = \begin{bmatrix} \lambda \\ \lambda v_2 \\ \lambda v_3 \end{bmatrix}.$$

**1.** Do the matrix multiplication and find $A\mathbf{n}$.

$$\begin{bmatrix} 0 & 1.5 & 9.0 \\ .1 & 0 & 0 \\ 0 & .8 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1.5v_2 + 9v_3 \\ .1 \\ .8v_2 \end{bmatrix} \longrightarrow A\mathbf{n} = \begin{bmatrix} 1.5v_2 + 9v_3 \\ .1 \\ .8v_2 \end{bmatrix}$$

**2.** The vector equation $A\mathbf{n} = \lambda\mathbf{n}$ leads to three scalar equations. What are they?

Equation 1 $1.5v_2 + 9v_3 = \lambda$

Equation 2 $.1 = \lambda v_2$

Equation 3 $.8v_2 = \lambda v_3$

**3.** Solve for $v_2$ and $v_3$ in terms of $\lambda$

Eq 2 $\Rightarrow$ $v_2 = \dfrac{.1}{\lambda}$

Eq 3 $\Rightarrow$ $v_3 = \dfrac{.8v_2}{\lambda} = \dfrac{.8(.1)}{\lambda^2}$

$v_2 = \dfrac{.1}{\lambda}$

$v_3 = \dfrac{.08}{\lambda^2}$

4. Find the equation satisfied by $\lambda$.

Now eq. 1 implies

$1.5 v_2 + 9 v_3 = \lambda$

so $\frac{1.5(.1)}{\lambda} + \frac{9(.08)}{\lambda^2} = \lambda$. Divide by $\lambda$ to get

Equation is $\frac{.15}{\lambda^2} + \frac{.72}{\lambda^3} = 1$

5. Use your calculator to solve this equation and find $\lambda$.

Let

$Y_1 = .15/X\text{^}2 + .72/X\text{^}3 - 1$

$\lambda =$ .95199

Xmin = .8

Xmax = 1.2

and do ZoomFit to get the graph.

NOW use 2nd calc zero to find $\lambda = .95199\ldots$