Mathematics 172

Quiz 18 Name: Key

You must show your work to get full credit.

For the Leslie matix

$$A = \begin{bmatrix} 0.0 & 2.5 & 12.0 \\ 0.2 & 0.0 & 0.0 \\ 0.0 & 0.25 & 0.0 \end{bmatrix}$$

1. Find the finite growth rate λ to three decimal places.

Let $\vec{N} = \begin{bmatrix} 1 \\ n_2 \\ n_3 \end{bmatrix}$ then

$\lambda =$ 1.0382

$$A\vec{N} = \begin{bmatrix} 0 & 2.5 & 12 \\ .2 & 0 & 0 \\ 0 & .25 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 2.5 n_2 + 12 n_3 \\ .2 \\ .25 n_2 \end{bmatrix}$$

If $\vec{N}$ is at stable age distribution

$$A\vec{N} = \lambda \vec{N} = \begin{bmatrix} \lambda \\ \lambda n_2 \\ \lambda n_3 \end{bmatrix}$$

2. Find the stable age distribution.

The proportion in Stage 1 is $\frac{1}{1.2390} = .807$

The proportion in Stage 2 is $\frac{.1926}{1.2390} = .155$

The proportion in Stage 3 is $\frac{.0464}{1.2390} = .037$

$A\vec{N} = \lambda \vec{N}$

leads to the 3 equations

① $2.5 n_2 + 12 n_3 = \lambda$

② $.2 = \lambda n_2$

③ $.25 n_2 = \lambda n_3$

From ② $n_2 = \frac{.2}{\lambda}$

use this in ③ to get

$$n_3 = \frac{.25}{\lambda} n_2 = \frac{(.25)(.2)}{\lambda^2} = \frac{.05}{\lambda^2}$$

Use these in ① to get

$$2.5\left(\frac{.2}{\lambda}\right) + 12\left(\frac{.05}{\lambda^2}\right) = \lambda$$

divide by λ

$$\frac{.5}{\lambda^2} + \frac{.6}{\lambda^3} = 1$$

solve (calcator) $\lambda = 1.0382$

so $n_2 = \frac{.2}{\lambda} = .1926$

$n_3 = \frac{.25(n_2)}{\lambda} = .0464$

$$\vec{N} = \begin{bmatrix} 1 \\ .1926 \\ .0464 \end{bmatrix}$$

$S = 1 + .1926 + .0464$
$= 1.2390$