Mathematics 172

Quiz 20 Name: Key

***You must show your work to get full credit.***

Our model for competing species is that if

$$x(t) = \text{Number of first species at time } t$$
$$y(t) = \text{Number of second species at time } t$$
$$K_1 = \text{Carrying capacity of first species}$$
$$K_2 = \text{Carrying capacity of second species}$$
$$r_1 = \text{Per capita growth rate of first species}$$
$$r_2 = \text{Per capita growth rate of second species}$$

then

$$\frac{dx}{dt} = r_1 x \left(\frac{K_1 - x - \alpha y}{K_1}\right)$$
$$\frac{dy}{dt} = r_2 y \left(\frac{K_2 - \beta x - y}{K_2}\right)$$

The equilibrium points for this are the points, $(x, y)$, where that make

$$\frac{dx}{dt} = \frac{dy}{dt} = 0.$$

Let's do an numerical example:

$$\frac{dx}{dt} = 0.2x \left(\frac{50 - x - .333y}{50}\right)$$
$$\frac{dy}{dt} = 0.5y \left(\frac{60 - .5x - y}{60}\right)$$

Find the equilibrium points.

$\frac{dx}{dt} = 0 \Rightarrow x = 0$ or $x + .333y = 50$

$\frac{dy}{dt} = 0 \Rightarrow y = 0$ or $.5x + y = 60$

so we get $x = 0, y = 0$; $(0,0)$

$x = 0$, $y = 0$; $(0, 60)$

$y = 0$, $x = 50$; $(50, 0)$

$x + .333y = 50$, $.5x + y = 60$

$\Rightarrow x = 40, y = 30$

The points are $(0,0), (50,0), (0,60), (40,30)$