Mathematics 172

Quiz 23 Name: Key

You must show your work to get full credit.

For the predator-prey system

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - sxy$$
$$\frac{dy}{dt} = -uy + vxy$$

where

x = size of prey population,
y = size of predator population,
r = per capita growth rate of x-species,
K = carrying capacity of x-species with no predators,
u = per capacity death rate of y-species without any prey,
s, v = constants that tell the rate of interaction between the two species.

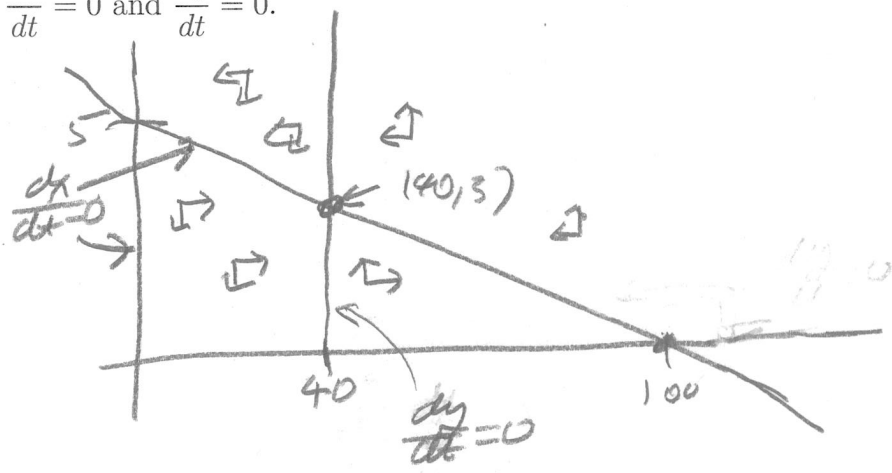
For the system

$$\frac{dx}{dt} = .1x\left(1 - \frac{x}{100}\right) - .02xy \quad = x\left(.1\left(1 - \frac{x}{100}\right) - .02y\right) = 0$$
$$\frac{dy}{dt} = -.4y + .01xy \quad = y(-.4 + .01x) = 0$$

1. Draw the phase plane showing where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

$.1\left(1 - \frac{x}{100}\right) - .02y = 0$
$x = 0 \Rightarrow y = \frac{.1}{.02} = 5$
so $(0, 5)$ is y-intercept
$y = 0 \Rightarrow x = 100$
$(100, 0)$ is x-intercept

$-.4 + .01x = 0$
$x = \frac{.4}{.01} = 40$



2. What are the rest points? Rest points are $(0,0)$, $(100,0)$, $(40,3)$

$(0,0)$, $(100,0)$ from the blobs. The lines cross where $x = 4$, so $.1\left(1 - \frac{40}{100}\right) - .2y = 0 \Rightarrow y = 3$

3. Draw in arrows in each region that show what directions the points are moving.