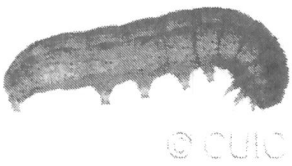
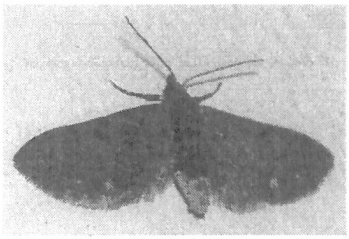
Mathematics 172

Quiz 29 Name: Key

You must show your work to get full credit.

In the Midwest there are large numbers of fields of oats, but they tend to be separated by huge number of fields of corn and soybeans. One of the pests of oats is the army worm (Mythimna unipuncta) which is the caterpillar of the white-speck moth.





Assume these breed once a year, that the probability an unpopulated field becomes populated the next year is .1, and that the probability a populated fields is unpopulated the next year is .8. Let f_t be the proportion of fields that are populated by army worms in year t where t is the number of years after 2,000.

1. Write the difference equation satisfied by f_t. Equation is $\Delta f = .1(1-f) - .8f$

2. What is the equilibrium point of this equation? The point is $f^* = .1111$

Solve $\Delta f = .1(1-f) - .8f = 0$

$.1 - .1f - .8f = 0$

$-.9f = -.1$

$f^* = \frac{-.1}{-.9} = .1111$

3. What percent of the fields will have arm worm populations in the long run?

Percentage is 11.11%

4. Assume in 2,000 that 30% of the fields are infested with army worms.

(a) What percent are infested in 2,002? Percentage is 11.30%

(b) What percent are infested in 2,010? Percentage is 11.11%

Write $\Delta f = .1(1-f) - .8f = -.9f + .1 = rf + s$

The solution is $f_t = f^* + (f_0 - f^*)(1+r)^t$

$= .1111 + (.3 - .1111)(1 - .9)^t$

$= .1111 + .1889(.1)^t$

In 2002 $t = 2$ $f_2 = .1111 + .1889(.1)^2 = .11299$ (11.30%)

In 2010 $t = 10$ $f_2 = .1111 + .1889(.1)^{10} = .1111$