Mathematics 172 Homework

The solution for these problems are after the last problem. Recall that an ***equilibrium point*** of the system is a point where both $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

1. For the system of differential equations

$$\frac{dx}{dt} = .4x\left(\frac{100 - x - .4y}{100}\right)$$
$$\frac{dy}{dt} = .6y\left(\frac{200 - .8x - y}{200}\right)$$

draw the phase plane (which for us is just a fancy term for the first quadrant of the x-y plane) showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

(d) Fill in arrows showing the direction that points are moving and use them to find which of the equilibrium points are stable.

2. For the system of differential equations

$$\frac{dx}{dt} = .35x\left(\frac{100.0 - x - 1.52y}{100.0}\right)$$
$$\frac{dy}{dt} = .07y\left(\frac{150.0 - 3.75x - y}{150.0}\right)$$

draw the phase plane showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

(d) Fill in arrows showing the direction that points are moving and use them to find which of the equilibrium points are stable.

3. For the system of differential equations

$$\frac{dx}{dt} = .33x\left(\frac{300.0 - x - 0.67y}{300.0}\right)$$
$$\frac{dy}{dt} = .51y\left(\frac{250.0 - 4.17x - y}{250.0}\right)$$

draw the phase plane showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

(d) Fill in arrows showing the direction that points are moving and use them to find which of the equilibrium points are stable.

4. For the system of differential equations

$$\frac{dx}{dt} = .023x\left(\frac{100.0 - x - 2.86y}{100.0}\right)$$
$$\frac{dy}{dt} = .1y\left(\frac{80.0 - 0.40x - y}{80.0}\right)$$

draw the phase plane showing

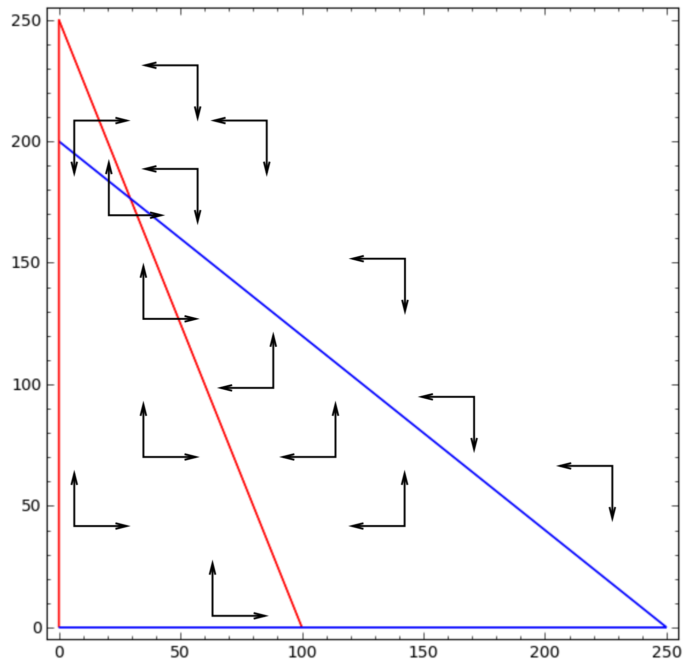
(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

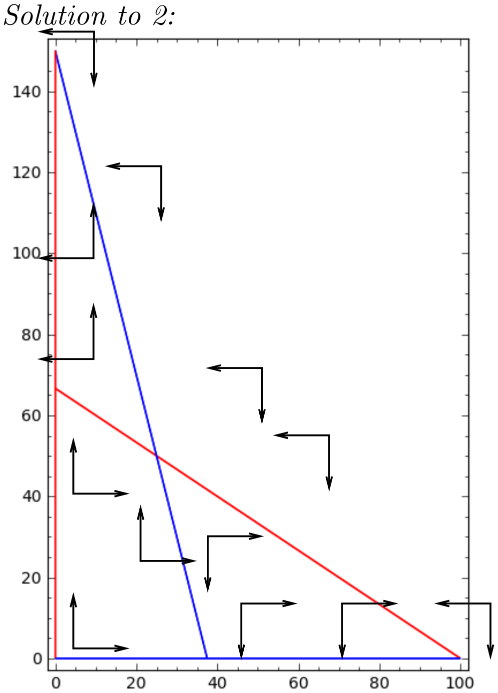
(d) Fill in arrows showing the direction that points are moving and use them to find which of the equilibrium points are stable.

Solution to 1:



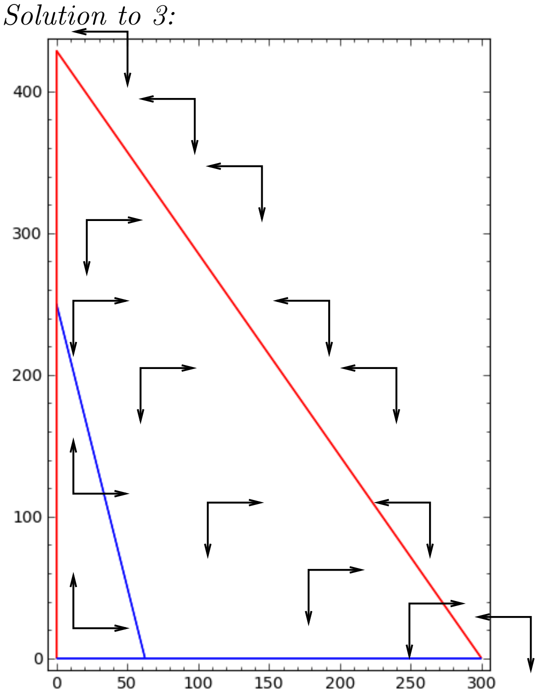
(a) The $dx/dt = 0$ lines are in red.
(b) The $dy/dx = 0$ lines are in blue.
(c) The equilibrium points are $(0, 0)$, $(100, 0)$, $(0, 200)$, and $(29.41, 176.5)$.
(d) The only stable equilibrium point is $(29.41, 176.5)$.

Solution to 2:



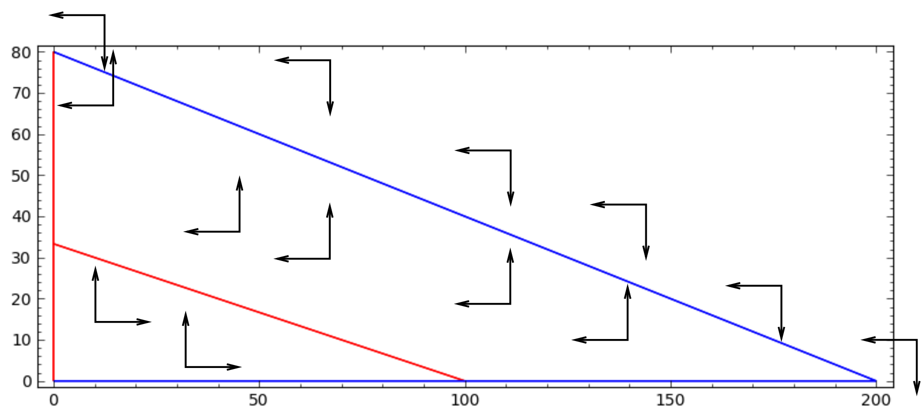
(a) The $dx/dt = 0$ lines are in red.
(b) The $dy/dx = 0$ lines are in blue.
(c) The equilibrium points are $(0,0)$, $(100,0)$, $(0,150)$, and $(27.2,47.9)$.
(d) The stable equilibrium points are $(100,0)$ and $(0,150)$.

Solution to 3:



(a) The $dx/dt = 0$ lines are in red.
(b) The $dy/dx = 0$ lines are in blue.
(c) The equilibrium points are $(0, 0)$, $(300, 0)$, and $(0, 250)$.
(d) The only stable equilibrium point is $(300, 0)$.

Solution to 4:



(a) The $dx/dt = 0$ lines are in red.
(b) The $dy/dx = 0$ lines are in blue.
(c) The equilibrium points are $(0, 0)$,$(100, 0)$, and $(0, 80)$.
(d) The only stable equilibrium point is $(0, 80)$.