Mathematics 172 Homework, January 22, 2018.

Example 1. A cell has volume $V = 8 \times 10^{-6}\text{mm}^3$ and surface area $A = 3.6 \times 10^{-3}\text{mm}^2$. Assume that oxygen, O_2, passes through the cell membrane at a rate of $.5(\text{mg/mm}^2)/\text{hr}$

(a) What is the total amount of O_2 that is comming into the cell per hour?

Solution:

$$\text{Total } O_2/\text{hour} = (3.6 \times 10^{-3}\text{mm}^2) \times .5(\text{mg/mm}^2)/\text{hr} = .0018\text{mg/hr}.$$

(b) What is the amount of O_2 per volume comming into the cell per hour?

Solution: Take the last answer and divide by the volume:

$$\text{Rate of } O_2 \text{ per volume} = \frac{.0018\text{mg/hr}}{8 \times 10^{-6}\text{mm}^3} = 225(\text{mg/mm}^2)/\text{hr}.$$

(c) If the cell needs $50(\text{mg/mm}^3)/\text{hr}$ of O_2 to survive, then how much can it be magnified before it dies from lack of oxygen?

Solution: Let λ be the factor by which it is magnified. Then by our rules for scaling we have we have

$$V_{mag} = 8 \times 10^{-6}\lambda^3\text{mm}^3, \qquad A_{mag} = 3.6 \times 10^{-3}\lambda^2\text{mm}^2$$

Thus

$$\text{Total } O_2 \text{ intake} = A_{mag} \times .5(\text{mg/mm}^2)/\text{hr} = .0018\lambda^2\text{mg/hr}$$

and

$$\text{Rate of } O_2 \text{ per volume} = \frac{.0018\lambda^2\text{mg/hr}}{8 \times 10^{-6}\lambda^3\text{mm}^3} = \frac{225(\text{mg/mm}^2)/\text{hr}}{\lambda}.$$

The threshold where oxygen starvation sets in is when

$$\text{Rate of } O_2 \text{ per volume} = 50(\text{mg/mm}^3)/\text{hr}.$$

That is

$$\frac{225(\text{mg/mm}^2)/\text{hr}}{\lambda} = 50(\text{mg/mm}^3)/\text{hr}.$$

Solving for λ gives

$$\lambda = \frac{225}{50} = 4.5$$

Therefore the cell can only grow to 4.5 times its length. □

1. A cell has volume $V = 4.6 \times 10^{-6}\text{mm}^3$ and surface area $A = 6.7 \times 10^{-3}\text{mm}^2$. Assume that oxygen, O_2, passes through the cell membrane at a rate of $.62(\text{mg/mm}^2)/\text{hr}$

(a) What is the total ammount of O_2 that is comming into the cell per hour? *Answer:* $4.154 \times 10^{-3}\text{mg/hr}$.

(b) What is the amount of O_2 per volume comming into the cell per hour? *Answer:* $903.04(\text{mg/mm}^2)/\text{hr}$.

(c) If the cell needs $377(\text{mg}/\text{mm}^3)/\text{hr}$ of O_2 to survive, then how much can it be magnified before it dies from lack of oxygen?

Answer: The magnification factor is $\lambda = 18.06$.