Mathematics 172 Homework, January 14, 2019.

What we saw in class today was

Proposition 1. *Let r be a constant. Then the solution to $P'(t) = rP(t)$ is*

$$P(t) = P(0)e^{rt}.$$

1. Find the solutions to the following:

(a) $P'(t) = 1.2P(t)$, $P(0) = 51$. Answer: $P(t) = 51e^{1.2t}$.

(b) $N'(t) = -.15N(t)$, $N(0) = 10.3$. Answer: $N(t) = 10.3e^{-.15t}$.

(c) $\dfrac{dA}{dt} = 2.1A$, $A(0) = 9$. Answer: $A(t) = 9e^{2.1t}$.

Here is an example of something a bit more complicated. We wish to solve

$$P'(t) = rP(t), \quad P(0) = 400, \quad P(2) = 412.$$

In this case r is unknown. We know that

$$P(t) = P(0)e^{rt} = 400e^{rt}.$$

We get anther equation

$$P(2) = 400e^{2r} = 412.$$

We can solve this to get

$$r = \ln(412/400)/2 = 0.01478$$

and thus

$$P(t) = 400e^{.1478t}$$

2. Solve the following:

(a) $P'(t) = rP(t)$, $P(0) = 51$, $P(3) = 62$. Answer: $P(t) = 51e^{.0651t}$.

(b) $N'(t) = aN(t)$, $A(0) = 97$, $A(10) = 85$. (a is a constant.) Answer: $N(t) = 97e^{-.01321t}$.

(c) $Q(t) = rQ(t)$, $Q(0) = 513$, $Q(1.3) = 520$. Answer: $Q(t) = 513e^{.0104t}$.

3. If $P'(t) = .15P(t)$ and $P(0) = 100$, then

(a) what is $P(5)$?

(b) how long until $P(t) = 500$?

Solution: We know that $P(t) = 100e^{.15t}$. So for part (a) just plug in $t = 5$, that is $P(5) = 100e^{.15(5)} = 211.7$.

For part (b) we need to solve $P(t) = 100e^{.15t} = 500$. The solution is $t = 10.73$.

4. If $N'(t) = -.05N(t)$ and $N(0) = 5.1$ then

(a) what is $N(20)$? Answer: $N(20) = 1.8761$.

(b) how long until $N(t) = .3$? Answer: $t = 56.664$.