Mathematics 172 Homework, January 29, 2019.

We have just derived the logistic equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

for the size, $P(t)$, of a population at time t given its intrinsic growth rate r and carrying capacity K. Let us do some variants on this model.

1. Assume that a population of algae is growing in an aquarium with an intrinsic growth rate of $r = .20$ (grams/gram)/day and that the carrying capacity of the aquarium is 800 grams of algae. Let $A(t)$ be the number of grams of algae in the aquarium on day t.

(a) Assuming that the algae grows logistically, what is the rate equation satisfied by A? *Solution:* This is the logistic equation, which is something that for the rest of the term you should have memorized,

$$\frac{dA}{dt} = .20A\left(1 - \frac{A}{800}\right).$$

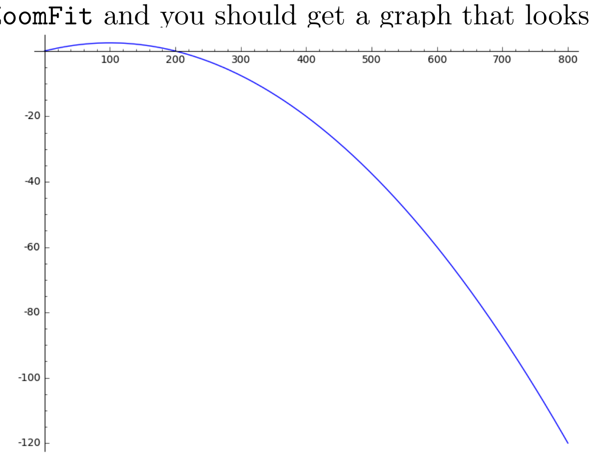
(b) Assume that the owner of the aquarium adds some fresh water shrimp that eat 15% of the algae per day. That is the new rate equation satisfied by A? *Solution:* It will be

$$\frac{dA}{dt} = .20A\left(1 - \frac{A}{800}\right) - .15A.$$

(c) What is the new carrying capacity for the algae? *Solution:* One way to do this would be to set $.20A\left(1 - \frac{A}{800}\right) - .15A = 0$ and solve for A. To avoid algebra we can also do this on the calculator. Put in

```
\Y1 = .2X(1-X/800) - .15 X
Xmin = 0
Xmax = 800
```

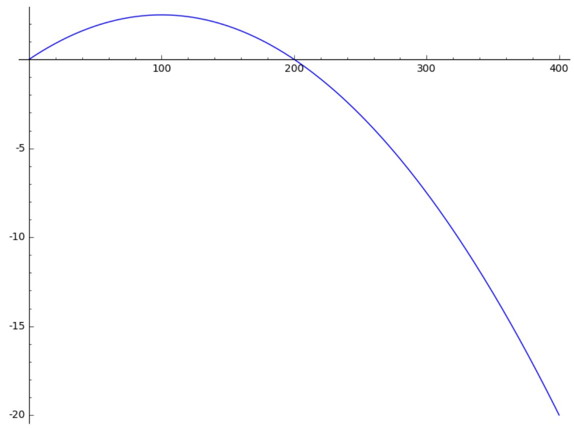
and do a `0: ZoomFit` and you should get a graph that looks like



Clearly one zero of this is $A = 0$, you can use the `2nd calc 2:zero` to get `X=200` for the other zero. Thus the equilibrium points are $A = 0$ and

$A = 200$. Now draw the pictures of $A(t)$ as we have been doing to see that $A = 200$ is stable. Thus $A = 200$ is the new carrying capacity.

Remark: The graph on your calculator map have been a bit hard to read because too much of it was below the axis. This can be fixed by redrawing it with a smaller window. There is clearly no zero larger than $A = 400$ so try the window `Xmin = 0` and `Xmax = 400` and the picture will look like



where it is easier to see where the zero is. You could even get by with setting `Xmax = 300` for this problem where it would be even easier to read.

2. An aquaculturist is rising tilapia. As a cheap source of fish food he has a pond growing duckweed. Assume the duckweed grows logistically with an intrinsic growth rate of $r = .8$(kg/kg)/day and a carrying capacity of $K = 30$ kg of duckweed. Let $W(t)$ be the wight of the duckweed in the pond after t days.

(a) What is the rate equation satisfied by $W(t)$? *Solution:*

$$\frac{dW}{dt} = .8W\left(1 - \frac{W}{30}\right)$$

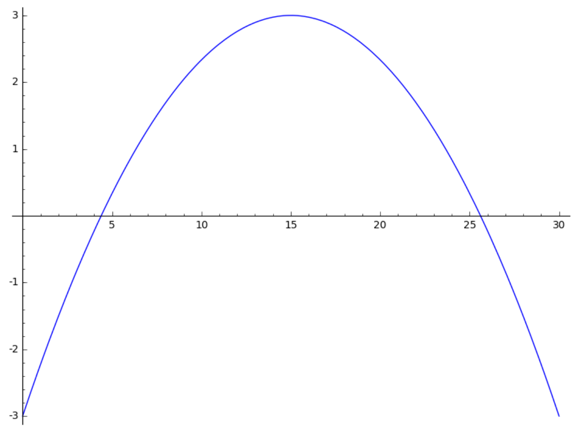
(b) The aquaculturist starts harvesting the duckweed at the rate of 3 kg/day. What is the new rate equation satisfied by $W(t)$? *Solution:* The equation is

$$\frac{dW}{dt} = .8W\left(1 - \frac{W}{30}\right) - 3.$$

(c) What are the equilibrium points of the new rate equation? This time use

```
\Y1 = .8X(1-X/30)-3
Xmin = 0
Xmax = 30
```

and the graph should look like:



Now use the calculator to find the zeros, which are $A = 4.3934$ and $A = 25.607$. These are the equilibrium points.

(d) Which of the equilibrium points are stable and which are unstable? *Solution:* $A = A = 4.3934$ is unstable and $A = 25.607$ is stable.

(e) What is the new carrying capacity of the duckweed population? *Solution:* It is the stable equilibrium points $A = 25.607$ kg of duckweed.