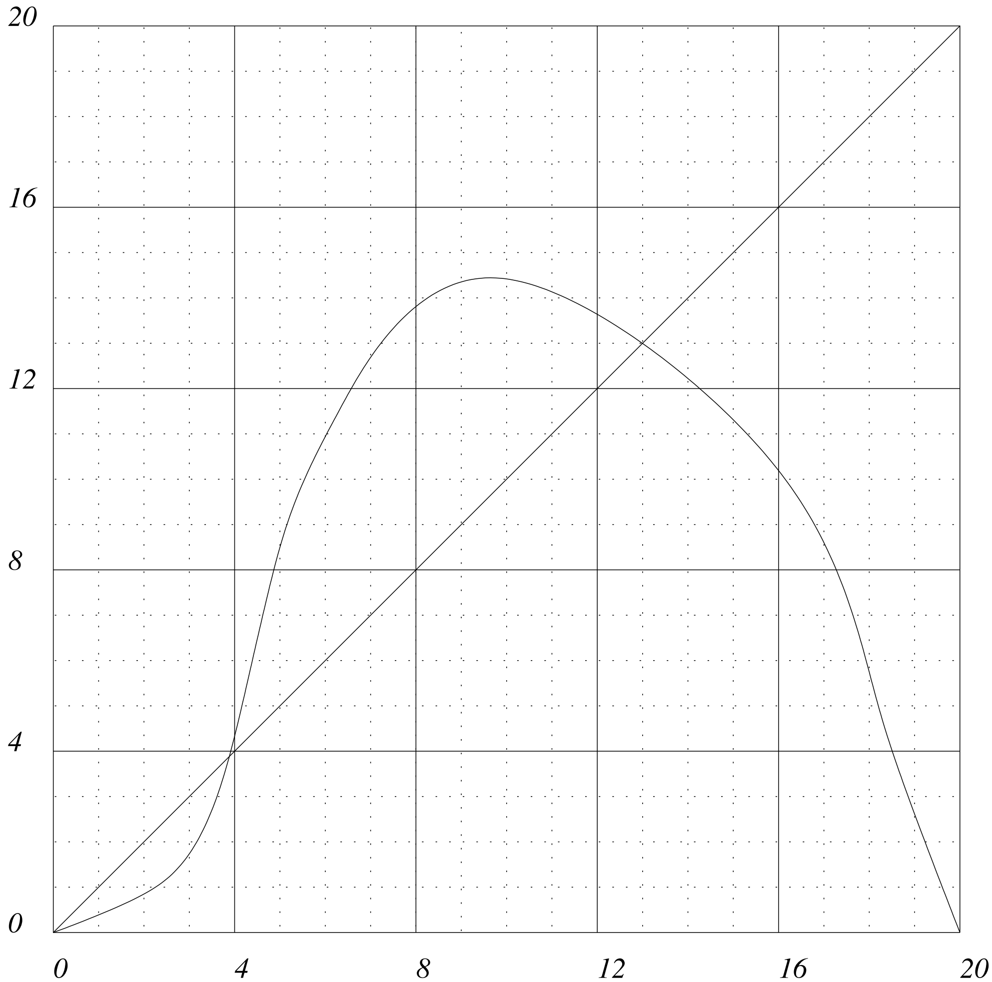
Mathematics 172 Homework, February 4, 2019.



The figure above gives the graph of the number, $N_{t+1} = f(N_t)$, of fish in a pond as a function of N_t. These fish set up territories and do not move far from their home base. Likewise if there are too many of them they are overcrowded and the population size goes down. Thus if there are too few fish they are spread too far apart to breed. Here we do a somewhat detailed analysis of this model. We will do this in the form of problems. The solutions to these problems are given after the end of the problem set.

1. (a) If $N_0 = 16$, estimate N_1, N_2, and N_3.

(b) If $N_0 = 3$, estimate N_1, N_2, and N_3.

(c) If $N_0 = 19$, estimate N_1, N_2, and N_3.

2. What are the equilibrium points? Which of them are stable?

3. (a) If $N_0 = 16$ estimate N_{30}.

(b) If $N_0 = 3$ estimate N_{45}.

(c) If $N_0 = 19$ estimate N_{100}.

4. If we release N_0 fish in the pond, explain what happens in the long run. (What happens will depend on the value of N_0.

Answer for problem 1. (a) The red shows the cobweb for $N_0 = 16$. We see

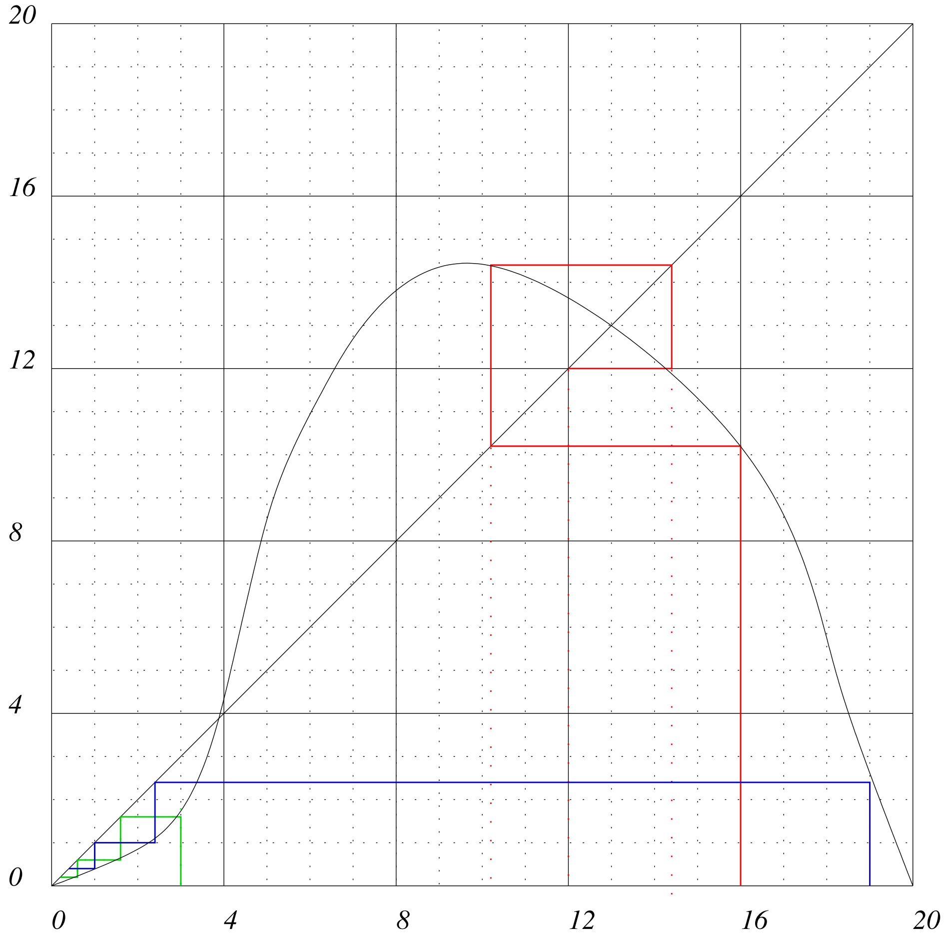
$$N_1 \approx 10.5 \qquad N_2 \approx 14.5 \qquad N_3 \approx 12$$

(b) The green shows the cobweb for $N_0 = 3$. This time:

$$N_1 \approx 1.6 \qquad N_2 \approx 0.5 \qquad N_3 \approx 0.2$$

(c) The blue shows the cobweb for $N_0 = 19$. This time:

$$N_1 \approx 2.5 \qquad N_2 \approx 1.0 \qquad N_3 \approx 0.4$$



Answer for problem 2. The equilibrium points are where the graph crosses the line $y = x$. There are three such points. They occur when $N_* = 0$, $N_* = 4$, and $N_* = 14$.

At $N_* = 0$, the slope of the graph is < 1 and therefore this point is i***stable***.

At $N_* = 4$, the slope of the graph is > 1 and therefore this point is ***untable***.

At $N_* = 14$, the slope is negative, by larger than -1, so $|\text{slope}| < 1$ and thus this point is stable.

Answer for problem 3. (a) Looking at the cobweb for problem 1 (a) we see that it will spiral into the point $(14, 14)$. Therefore $N_{30} \approx 14$. (In fact $N_t \approx 14$ for any large t).

(b) Using the cobweb for problem 1 (b) we see that it steps to to $(0, 0)$ and therefore $N_{45} \approx 0$. (And for any large t we have $N_t \approx 0$.)

(c) Using the cobweb for problem 1 (c) we see that it steps down to $(0, 0)$. Therefore for any large t we have $N_t \approx 0$. In particular $N_{100} \approx 0$.

Answer to problem 4. If $N_0 < 4$, then we can draw the cobweb and see that it steps down to $(0, 0)$. Therefore in this case the fish will out.

If $4 < N_0 < 18.5$, then the cobweb will spiral into $(14, 14)$ and so in this case after a few years the popular will settle down to a stable size of 14 fish.

If $18.5 < N_0 \leq 20$. Then the first step in the cobweb will have $N_1 < 4$. And from there the cobweb steps down to $(0, 0)$. So in this case the population also dies off.

In summary: If $N_0 < 4$ or $N_0 > 18.5$ the population dies off. If $4 < N_0 < 18.5$, then the population settles down to a stable size of $N_* = 14$ fish.