Mathematics 300

Quiz 18 Name: Key

You must show your work to get full credit.

We have proven the following severe times.

Proposition 1. *For any integers n, if n^2 is even, then n is even.*

1. Use this proposition to prove $\sqrt{2}$ is irrational.

Toward a contradiction assume $\sqrt{2}$ is rational. Then

$$\sqrt{2} = \frac{p}{q}$$

where p, q are integers and we can assume this fraction is in lowest terms. Squaring this equation gives

$$2 = \frac{p^2}{q^2}$$

so

$$(*) \qquad 2q^2 = p^2$$

This shows $p^2 = 2q^2$ and q^2 is an integer. so by Proposition 1 p is even. Thus $p = 2m$ for some integer m. Use this in equation $(*)$ to get

$$2q^2 = (2m)^2 = 4m^2$$

Divide by 2

$$q^2 = 2m^2$$

So q^2 is 2 times an integer, thus q^2 is even. Thus by the proposition q is even so $q = 2n$ for some integer n. Then

$$\frac{p}{q} = \frac{2m}{2n}$$ is not in lowest

terms, a contradiction.

2. Prove: If β is irrational, then so is $\dfrac{\beta+2}{\beta-1}$.

Towards a contradiction assume $\dfrac{\beta+2}{\beta-1}$ is rational. Then

$$\frac{\beta+2}{\beta-1} = \frac{m}{n}$$

for some integers m and n. Cross multiply and then do some algebra to get

$$n(\beta+2) = m(\beta-1)$$

$$n\beta + 2n = m\beta - m$$

$$n\beta - m\beta = -m - 2n$$

$$(n-m)\beta = -m-2n$$

$$\beta = \frac{-m-2n}{n-m} = \frac{p}{q}$$

where $p = -m-2n$, $q = n-m$ are integers. This implies β is rational, contradicting that it is irrational.