Mathematics 300

Quiz 1 **Name:** Key

You must show your work to get full credit.

1. Give our official definitions of the following:

(a) n is a even integer.

$n = 2q$ for some integer q.

(b) n is an odd integer.

$n = 2q + 1$ for some integer q.

2. Prove or give a counter example: If x and y odd, then $x^2 + 3y^3$ is odd.

This is False. If $x = y = 1$,

$x^2 + 3y^3 = 1^2 + 3(1)^3 = 4$

which is even. Thus

$x = 1$, $y = 1$ is a counterexample

3. Prove or give a counter example: If x is odd, then $x^2 + 1$ is even.

If x is odd, then $x = 2q + 1$ for some integer q. Then

$$x^2 + 1 = (2q+1)^2 + 1 = 4q^2 + 4q + 1 + 1 = 4q^2 + 4q + 2$$
$$= 2(2q^2 + 2q + 1).$$

To be a little more formal.

Step	Known	reason.
P	x is odd integer	Hypothesis
P_1	$x = 2q + 1$ for some integer q	Def. of odd
P_2	$x^2 + 1 = (2q+1)^2 + 1 = 2(2q^2 + 2q + 1)$	substitution and algebra
P_3	$x^2 + 1 = 2k$ for some integer k	use $k = 2q^2 + 2q + 1$
P_4	$x^2 + 1$ is even.	Definition of even.