Mathematics 300

Quiz 16

Name: Key

You must show your work to get full credit.

1. Prove: If n is an integer and n^2 is even, then n is even.

We prove the contrapositive: If n is odd, then n^2 is odd. Assume n is odd, then $n = 2q+1$ for some integer q. Then

$$\begin{aligned} n^2 &= (2q+1)^2 \\ &= 4q^2+4q+1 \\ &= 2(2q^2+2q)+1 \\ &= 2m+1 \end{aligned}$$

where $m = 2q^2+2q$ is an integer. This shows n^2 is odd. done

2. Use Problem 1 to show $\sqrt{2}$ is irrational.

Towards a contradiction assume $\sqrt{2}$ is rational. Then

$$(*) \quad \sqrt{2} = \frac{m}{n}$$

where m and n are integers and $n \neq 0$. We also assume this fraction is in lowest terms.

Multply $(*)$ by n

$$\sqrt{2}n = m$$

and square this to get

$$(**) \quad 2n^2 = m^2$$

Thus m^2 is two times an integer and thus is even. By problem 1 this implies m is even. Say $m = 2p$ for some integer p. Use this in $**$

$$2n^2 = (2q)^2 = 4q^2$$

$$n^2 = 2q^2$$

Thus n^2 is even and (problem 1 again) so n is even. Say $n = 2q$. Then

$$\frac{m}{n} = \frac{2p}{2q}$$

contradicting that $\frac{m}{n}$ is in lowest terms. done