Mathematics 300

Quiz 22 Name: Key.

You must show your work to get full credit.

Proposition. *For any integers n, if 7 divides n^2, then 7 divides n.* ☐

1. Use this proposition to prove $\sqrt{7}$ is irrational.

Towards a contradiction assume $\sqrt{7}$ is rational. Then

(*) $\sqrt{7} = \frac{a}{b}$

with $a, b \in \mathbb{Z}$. We can assume this is in lowest terms. Square (*) to get

$$7 = \frac{a^2}{b^2}$$

multiply by b^2.

$$7b^2 = a^2$$

This shows a^2 is 7 times the integer b^2 so $7 | a^2$. By the proposition this implies $7 | a$. Thus

$$a = 7p$$

for some integer p. Use this in $7b^2 = a^2$ to get

$$7b^2 = (7p)^2 = 49p^2$$

so $b^2 = 7p^2$

Thus b^2 is 7 times an integer and thus $7 | b^2$. By the proposition this gives $7 | b$. So $b = 7q$ for some integer q.

Therefore

$$\frac{a}{b} = \frac{7p}{7q}$$

contradicting that $\frac{a}{b}$ is in lowest terms.

done.