Mathematics 300

Quiz 28 Name: Key

You must show your work to get full credit.

1. Give a precise statement of the ***division algorithm***. For all integers a, b with $b > 0$, there are unique integers q, r such that

$$a = qb + r \quad \text{and} \quad 0 \le r < b$$

2. Assume that the local fast food restaurant only sells chicken nuggets in packages of size with 3 nuggets or 4 nuggets. For what values of n is possible to buy packages totaling exactly n nuggets? Prove your answer. We first make a table for small values. $\longrightarrow$

n	possible	
1	No	
2	No	
3	Yes	(3)
4	Yes	(4)
5	No	
6	Yes	2(3)
7	Yes	(3)+(4)
8	Yes	3(3)
9	Yes	3(3)
10	Yes	2(3)+(4)

From the table it looks like we can get exactly n nuggets for any n other than 1, 2, 5.

__Proposition__ If $n \ne 1, 2, 5$, then we can buy exactly n chicken nuggets.

__Base cases__ We see this is true for $n \le 9$ by the table.

__Induction hypothesis.__ $k \ge 9$ and we can buy exactly k nuggets.

__Induction goal:__ we can buy exactly $(k+1)$ nuggets.

__Case 1__ When we buy k nuggets there is a pack with 3 nuggets. Swap this package for one with 4 nuggets. Then we have $k - 3 + 4 = k+1$ nuggets. Goal achieved

__Case 2__ All the packages have 4 nuggets. Then swap 2 of these (possible as $k \ge 8$) for 3 packages of size 3. Then we have $k - 2(4) + 3(3) = k+1$ and our goal is achieved.

done