Mathematics 300

Quiz 29 Name: Key

You must show your work to get full credit.

1. Give the precise definition of $a \equiv b \pmod{n}$.

$n > 0$ and $n \mid (a-b)$

2. Let s_n be a sequence defined by

$$s_{n+1} = s_n + 2n + 5, \qquad s_0 = 0.$$

Use induction to prove $s_n = n(n+4)$

Base case $n = 0$, then $s_0 = 0 = 0(0+4)$ holds

Induction hypothesis: $s_k = k(k+4)$

Induction goal: $s_{k+1} = (k+1)((k+1)+4)$

i.e. $s_{k+1} = (k+1)(k+5)$.

Assume $s_k = k(k+4)$.

Then $s_{k+1} = s_k + 2k + 5$

$= k(k+4) + 2k + 5$

$= k^2 + 4k + 2k + 5$

$= k^2 + 6k + 5$

$= (k+1)(k+5)$

and we have completed the induction. done