Mathematics 300

**Quiz 30** Name: Key

***You must show your work to get full credit.***

**1.** Give a precise statement of the *division algorithm*.

For any integers $a, b$ with $b > 0$, there are unique integers $q, r$ such that

$a = qb + r$ and $0 \le r < b$

**2.** Define a sequence by $a_0, a_2, a_2, \ldots$ by

$$a_{n+1} = \frac{4}{5}a_n + 10, \qquad a_0 = 20$$

(a) Compute $a_1$ and $a_2$.

$a_1 =$ 26 $\qquad a_2 = \frac{154}{5} = 30.8$

$a_1 = \frac{4}{5}(20) + 10 = 4 \cdot 4 + 10 = 26$, $a_2 = \frac{4}{5} \cdot 26 + 10$

$= \frac{104}{5} + \frac{50}{5} = \frac{154}{5}$

(b) Prove $a_n < 50$ for all $n$.

Base case: $a_0 = 20 < 50$ holds,

Induction hypothesis: $a_k < 50$

Induction Goal: $a_{k+1} < 50$

Induction step: $a_{k+1} = \frac{4}{5} a_k + 10$

$< \frac{4}{5}(50) + 10$ as $a_k < 50$

$= 40 + 10$

$= 50.$