Mathematics 300

Quiz 8 Name: Key

You must show your work to get full credit.

1. Let a, b, and n be integers with $n \neq 0$. Define $a \equiv b \pmod{n}$.

$a \equiv b$ mod n if and only if $n \mid (b-a)$.

2. Prove or give a counterxample: If $a^2 \equiv 0 \pmod{4}$, then $a \equiv 0 \pmod{4}$.

This is false. A counterexample is $a = 2$.

Then $a^2 = 4 \equiv 0$ mod 4,

but $a = 2 \not\equiv 0$ mod 4

3. Prove: If $a \equiv 3 \pmod{9}$ and $b \equiv 4 \pmod{9}$, then $a + b \equiv 7 \pmod{9}$.

Assume $a \equiv 3$ mod 9 and $b \equiv 4$ mod 9.

Then $9 \mid (a-3)$ and $a \mid (b-4)$

So there are integers m, n with

$a - 3 = 9m, \quad b - 4 = 9n.$

By algebra

$a = 9m + 3 \quad b = 9n + 4.$

Thus

$$\begin{aligned} a + b - 7 &= 9m + 3 + 9n + 4 - 7 \\ &= 9(m+n) \\ &= 9q \end{aligned}$$

where $q = m + n$ is an integer by closure properties. Thus $9 \mid (a+b-7)$ and therefore

$a + b \equiv 7$ mod 9.