Mathematics 300

Quiz 9 Name: Key

You must show your work to get full credit.

1. Prove: If $x \equiv y \pmod{m}$ and $u \equiv v \pmod{m}$, then $xu \equiv yv \pmod{m}$.

Assume $x \equiv y \mod m$ and $u \equiv v \mod m$

Then $m \mid (x-y)$ and $m \mid (u-v)$

so there are integers p, q with

$x - y = pm, \quad u - v = qm$

by algebra

$x = y + pm \quad u = v + qm.$

Therefore

$xu = (y+pm)(v+qm)$

$= yv + yqm + vpm + pqm^2$

$= yv + m(yq + vp + pqm)$

$= yv + mr$

where $r = yq + vp + pqm$ is an integer by closure properties.

Thus $m \mid (xu - yv)$. That is $xu \equiv yv \mod m$

2. Prove or give a counterexample: If n is odd, then $3n^2 \equiv 3 \pmod{12}$.

<u>Proof</u> Assume n is odd. Then

$n = 2q + 1$

for some integer q. Then

$3n^2 = 3(2q+1)^2$

$= 3(4q^2 + 4q + 1)$

$= 12q^2 + 12q + 3$

$= 3 + 12(q^2 + q)$

$= 3 + 12p$

where $p = q^2 + q$ by closure properties.

Thus $3n^2 - 3 = 12p$

so $12 \mid (3n^2 - 3)$. That is

$3n^2 \equiv 3 \mod 12$