Mathematics 122

Quiz 20

Name: Key

You must show your work to get full credit.

1. Let u and v be functions of x and c, a and n constants.

(a) State the product rule for the derivative of the product $y = u(x)v(x)$.

$y' =$ $u'(x)v(x) + u(x)v'$

(b) State the quotient rule for $y = \dfrac{u(x)}{v(x)}$.

$y' =$ $\dfrac{u'(x)v - u(x)v'(x)}{v(x)^2}$

(c) State the power rule for the derivative of $y = cu(x)^n$

$y' =$ $cnu(x)^{n-1}u'(v)$

(d) State the rule for the derivative of $y = ce^{u(x)}$

$y' =$ $ce^{u(x)}u'(x)$

(e) State the rule for the derivative of $y = ca^{u(x)}$

$y' =$ $c\ln(a)e^{u(x)}u'(x)$

(f) State the rule for the derivative of $u = c\ln(x)$

$y' =$ $\dfrac{c}{x}$

(g) Let $g(x)$ be anther function. State the chain rule for the derivative of $y = g(u(x))$

$y' =$ $g'(u(x))u'(x)$

2. Let a, b, c and n be constants. Compute the following derivatives.

(a) $f(x) = x^2 - 3x + 7$ $\qquad f'(x) =$ $2x-3$

(b) $w = x^n e^{az}$ $\qquad \frac{dw}{dz} =$

$\cancel{w' = nx^{n-}}$

(c) $y = \pi x^e + \underbrace{e^3 + 2e - 5}_{\text{constant}}$ $\qquad \frac{dy}{dx} =$ $e\pi x^{e-1}$

$y' = e\pi x^{e-1} + 0$

ok here

(d) $y = x^2 e^{3x}$ $\qquad y' =$ $(2x+6x^2)e^{3x}$

$y' = 2x\, e^{3x} + x^2(3e^{3x})$

$= (2x + 6x^2)e^{3x}$

ok here

(e) $y = x^n e^{ax}$ $\qquad y' =$ $(nx^{n-1} + ax^n)e^{ax}$

$y' = nx^{n-1}e^{ax} + x^n a e^{ax}$

$= (nx^{n-1} + ax^n)e^{ax}$

(f) $A = 2\sqrt{r^2 + 4r}$ $= 2(r^2+4r)^{\frac{1}{2}}$ $\qquad \frac{dA}{dr} =$ $\frac{2r+4}{\sqrt{r^2+4r}}$

$\frac{dA}{dr} = 2(\frac{1}{2})(r^2+4r)^{-\frac{1}{2}}(2r+4) \leftarrow$ ok here

$= \frac{2r+4}{\sqrt{r^2+4r}}$

(g) $g(t) = \frac{t^2}{t^3+1}$ $\qquad g'(t) =$ $\frac{-t^4+2t^2}{(t^3+1)^2}$

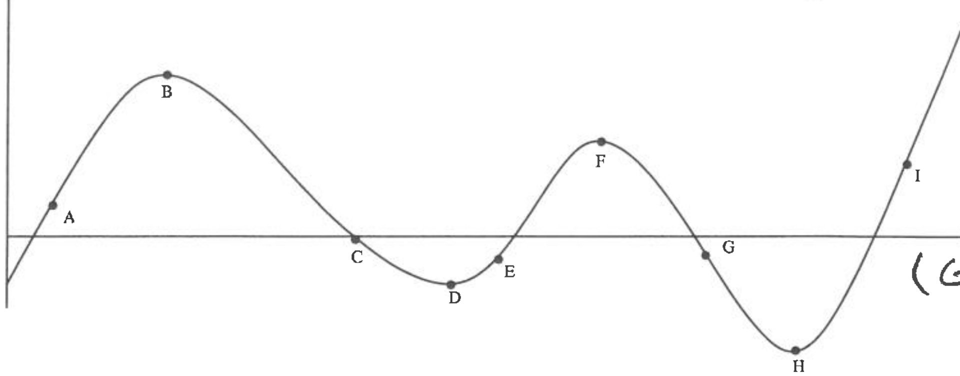
ok here

$g'(t) = \frac{2t(t^3+1) - t^2(3t^2)}{(t^3+1)^2}$

$= \frac{2t^4 + 2t^2 - 3t^4}{(t^3+1)^2}$

$= \frac{-t^4+2t^2}{(t^3+1)^2}$

3. For the labeled points on the figure on the following graph of $y = f(x)$



(G is inflection point.)

is $f(x) < 0$ D, E

is $f'(x) > 0$ A, E, I

is the function concave down A, B, F

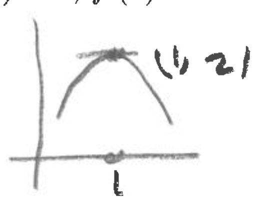
is a local maximum of f B, F

4. Draw graphs of a function

(a) That is increasing at a decreasing rate.

slope is positive (so increasing) and getting smaller (so rate of increase is decreasing)

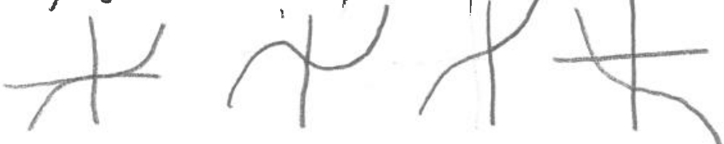
(b) With $f(1) = 2$, $f'(1) = 0$ and $f''(x) < 0$.



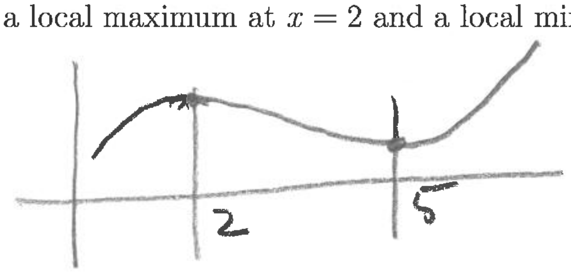
concave down

(c) That has an inflection point on the y-axis.

Any of the following work



(d) That has a local maximum at $x = 2$ and a local minimum at $x = 4$.



5. Let $f(x) = x^2(6a - x)^3$ where a is a constant.

(a) Find $f'(x)$ and write the result in factored form.

$f'(x) =$ $x(6a-x)^2(12a-5x)$

$$f'(x) = 2x(6a-x)^3 + x^2 3(6a-x)^2$$
$$= x(6a-x)^2(2(6a-x)+3x)$$
$$= x(6a-x)^2(12a-5x)$$

(b) What are the critical points of f? The critical points are $0,\ 6a,\ \frac{12a}{5}$

Solve
$f'(x) = x(6a-x)^2(12a-5x) = 0$
$x = 0,\quad 6a - x = 0 \Rightarrow x = 6a,\quad 12a - 5x = 0 \Rightarrow x = \frac{12a}{5}$

(c) On the interval $0 \le x \le 6a$ what are

The maximizer of $f(x)$ $\frac{12a}{5}$

The maximum of $f(x)$ $\frac{(12)^2(18)^3 a^5}{5^5}$

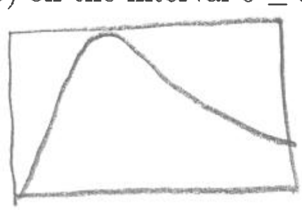
$f(0) = f(6a) = 0$
and $f(x) > 0$ on
$0 \le x \le 6a$

The only critical point with $0 < x < 6a$ is $x = \frac{12a}{5}$ which is thus maximizer.

The maximum is
$$f\left(\frac{12a}{5}\right) = \left(\frac{12a}{5}\right)^2\left(6a - \frac{12a}{5}\right)^3$$
$$= \frac{(12)^2a^2}{5^2}\left(\frac{30a-12a}{5}\right)^3$$
$$= \frac{(12)^2(18)^3}{5^5}a^5$$

6. Let $f(x) = 3x(1.5)^{-x+1}$.
Use your calculator to plot $y = f(x)$ on the interval $0 \le x \le 10$ and make a sketch of the result here.

$Y1 = 3X(1.5)^{-X+1}$
Xmin $= 0$
Xmax $= 10$
ZoomFit

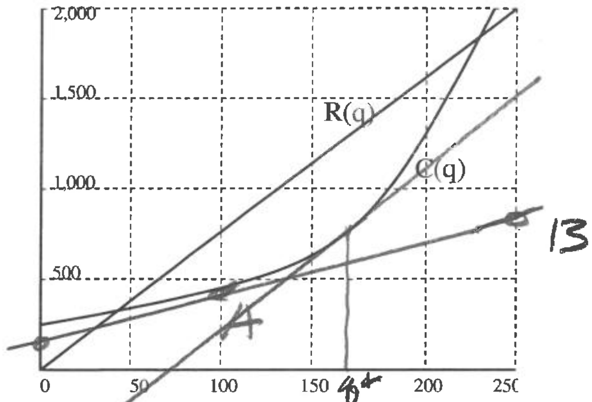


(a) Use your calculator to find

Maximizer of f on the interval 2.4663 Maximum of f on the interval 4.0829

By looking at graph
Minimizer of f on the interval 0 Minimum of f on the interval 0

(b) Say what you did on the calculator to find maximum.
2nd calc 4: maximum. Then left bound, right bound
Guess, to get $X = 2.4663020$, $Y = 4.082685$



7. The graph above gives the revenue, $R(q)$, and cost, $C(q)$, of producing q widgets.

(a) What is the revenue of selling a single widget. Revenue is $8/ wiget.

This is slope of Revenue line $= \dfrac{2000-0}{250-0} = \dfrac{2000}{250} = 8$

(b) Estimate the cost of producing the 101st widget. HINT: This is the same as estimating the marginal revenue $MC(100) = C'(100) =$ slope of tangent line when $q = 100$

$C'(100) \approx$ 1.666 (which is likely too small)

I am using the points $A = (100, 450)$, $B = (250, 700)$

slope $= \dfrac{700-450}{250-100} = 1.666$

(c) If 100 widgets are being produced, should production be increased or decreased and why?

The profit on producing one more is $8 - 1.666 > 0$. So we should produce another. Yes

(d) Estimate the number of widgets that should be sold to maximize the profit.

$q \approx$ 165

This is where tangent line to $C(q)$ curve is parallel to $R(q)$ line. It looks to be about q^* (on graph)

$q^* \approx 165$

8. The revenue brought in of selling q widgets is

$$R(q) = 50q$$

(that is they are being sold for \$50 each) and the cost of producing q widgets is

$$C(q) = 30 + .05q^2$$

How many widgets should be produced to maximize the profit.

The maximizer is $q =$ 500

The profit is

$$\pi(q) = R(q) - C(q)$$
$$= 50q - 30 - .05q^2$$

Method 1. The critical point is found by solving

$$\pi'(q) = 50 - 2(.05)(q)$$
$$= 50 - .1q = 0$$
$$-.1q = -50$$
$$q = \frac{50}{.1} = 500$$

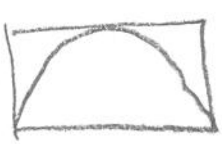
This is maximizer. So

Method 2 Plot $\backslash Y1 = 50X - 30 - .05X^2$

Xmin = 0

Xmax = 1000 (found by trial and error)

ZoomFit



2nd calc 4: Maximum to get

$X = 500.$ $\quad Y = 12470$

maximizer.