# Mathematics 554 Homework.

In what follows *Vector Calculus* refers to the text of *Vector Calculus* by Michael Corral.

**Problem** 1. Let $\mathbf{a}$ and $\mathbf{b}$ be the vectors

$$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} \qquad \mathbf{b} = -\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

Compute the following

(a) $3\mathbf{a} - 2\mathbf{b}$
(b) $\mathbf{a}\cdot\mathbf{b}$ *Hint:* for the ***dot product*** aka ***scalar product*** see *Vector Calculus* page 15.
(c) $\mathbf{a} \times \mathbf{b}$ *Hint:* for the ***cross product*** see *Vector Calculus* page 20.
(d) $\|\mathbf{a}\|$ *Hint:* For the ***length*** also called the ***norm*** or ***magnitude*** of a vector see *Vector Calculus* page 7.
(e) The distance between $\mathbf{a}$ and $\mathbf{b}$. *Hint:* for the distance between vectors see *Vector Calculus* page 7. Note this can be written more succinctly as $\|\mathbf{a} - \mathbf{b}\|$.
(f) The midpoint between $\mathbf{a}$ and $\mathbf{b}$. *Hint:* The midpoint between two numbers $x$ and $y$ is $\frac{1}{2}(x + y)$. The same idea works for vectors.
(g) A parametric equation of the line line between $\mathbf{a}$ and $\mathbf{b}$. *Hint:* For lines through a pair of points see *Vector Calculus* page 31.

**Problem** 2. Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$.

(a) What is the velocity vector to $\mathbf{r}$? *Hint:* The velocity vector is just the derivative $\mathbf{r}'(t)$.
(b) What is the tangent line to $\mathbf{r}$ when $t = -1$. *Hint:* Compute $\mathbf{r}(-1)$ and $\mathbf{r}'(-1)$. Then we just want the line through $\mathbf{r}(-1)$ in the direction of $\mathbf{r}'(-1)$. This brings us back to *Vector Calculus* page 31 (equation 1.16).
(c) Write the integral for the equation of the length of $\mathbf{r}$ between $t = -1$ and $t = 2$? *Hint:* For the length of a curve see *Vector Calculus* page 59. Just set up the integral, you are not being ask to evaluate it.
(d) At what points does this curve intersect the plane with equation $3x - y = 2$? *Hint:* Here is a related example. Let $\mathbf{f}(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$. Let us find the points where this intersects the plane $2x - 4y + z = -1$. We write $\mathbf{f}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ where $x(t) = 1$, $y(t) = t$ and $z(t) = t^2$. For $\mathbf{f}(t)$ to be on the plane $2x - 4y + z = -1$ we have

$$2x(t) - 4y(t) + z(t) = 2(1) - 4(t) + t^2 = -1,$$

which can be rearranged as

$$t^2 - 4t + 3 = 0.$$

This factors as $(t - 3)(t - 1) = 0$. So $t = 1$ and $t = 3$. Thus the points of intersection are

$$\mathbf{f}(1) = \mathbf{i} + \mathbf{j} + \mathbf{k} = (1, 1, 1), \quad \text{and} \quad \mathbf{f}(3) = \mathbf{i} + 3\mathbf{j} + 9\mathbf{k} = (1, 3, 9).$$

**Problem** 3. What is an equation for the plane through $(1, 0, 0)$, $(1, 2, 0)$ and $(1, 2, 3)$? *Hint:* For the equation of a plane through three point see *Vector Calculus* page 36.

**Problem** 4. Let a $z = x^2 + 4x + y^2 - 6y$.

(a) Compute the partial derivative $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

(b) What is an equation of the tangent plane to $z = x^2 + 4x + y^2 - 6y$ at the point where $x = 3$ and $y = -1$. *Hint:* For the equation of a tangent plane see *Vector Calculus* Section 2.3 page 75.

(c) What is the minimum value of $z$ and where does it occur? *Hint:* For finding maximums and minimums of functions of two variables see *Vector Calculus* Section 2.5 page 83. But for for the problem here you can get by with high school algebra (completing the square).