Mathematics 554 Homework.

This is a review problem set of the test.

Problem 1. You should be able to set up and (if the integrals are not two complicated) evaluate line integrals. For practice in *Vector Calculus*Problems 1, 3, 5, 7, 9, 11 in Section 4.1 Page 142 have answers in the back of the book.

Problem 2. You should know the proof of Green's Theorem as presented on Homework 4.

Problem 3. For practice in using Green's Theorem to evaluate line integrals look at Problems 1, 3 on page 155 of the *Vector Calculus*which have answers in the back of the book.

Problem 4. For practice in finding potentials of conservative vector fields look at Problems 5, 7 on page 155 of *Vector Calculus*which have answers in the back of the book.

Problem 5. You should also know the chain rule. Here are some forms of this result.

(a) If $f(x, y)$ is a differentiable function and x and y are functions of t. Then

$$\frac{d}{dt} f(x(t), y(t)) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}.$$

(b) This can be written in more compact notation using the ***gradient***, ∇f, which is

$$\nabla f \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}.$$

Then if $\mathbf{r}(x) = (x(t), y(t))$ the chain rule can be written as

$$\frac{d}{dt}\nabla f(\mathbf{r}(t)) \boldsymbol{\cdot} \mathbf{r}'(t).$$

(c) Note that if $\mathbf{r}\colon [a, b]\colon \mathbb{R}^2$ parameterizes the curve C, then the chain rule can be combined with the Fundamental Theorem of Calculus to show

$$\begin{aligned}
\int_{\mathcal{D}} \nabla f \boldsymbol{\cdot} d\mathbf{r} &= \int_a^b \nabla f(\mathbf{r}(t)) \boldsymbol{\cdot} \mathbf{r}'(t)\, dt \\
&= \int_a^b \frac{d}{dt} f(\mathbf{r}(t))\, dt \\
&= f(\mathbf{r}(b)) - f(\mathbf{r}(a)).
\end{aligned}$$

Problem 6. You will have to set up and evaluate double and triple integrals. In *Vector Calculus*Problems 1, 3, 5, 6, 7, 9, and 10 in Section 3.2 (page 155) and Problems 1, 3, 5, 7, 10 in Section 3.3 (page 112) have the answers in the back of the book.